23. $\int_{-\infty}^{\infty} \frac{dx}{e^x+e^{-x}} = \int_{-\infty}^{0} \frac{dx}{e^x+e^{-x}} + \int_{0}^{\infty} \frac{dx}{e^x+e^{-x}}$

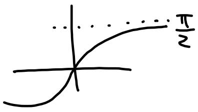
converges to $\frac{\pi}{2}$

$\frac{\pi}{4}$ $\frac{\pi}{4}$

$\lim_{b\to\infty} \int_0^b \frac{e^x\,dx}{e^x\left(e^x+\frac{1}{e^x}\right)} = \lim_{b\to\infty} \int_0^b \frac{e^x\,dx}{(e^x)^2+1}$

$u = e^x$

$du = e^x\,dx$



$\int \frac{du}{u^2+1} = \tan^{-1} u = \tan^{-1} e^x$

$\lim_{b\to\infty} \tan^{-1} e^x \Big|_0^b = \lim_{b\to\infty} \tan^{-1} e^b - \tan^{-1} 1$

$\frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

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25 $\int_0^2 \frac{dx}{1-x^2}$ undefined at $x = 1$

diverges

$\int_0^1 \frac{dx}{1-x^2} + \int_1^2 \frac{dx}{1-x^2} = \lim_{b\to 1} \int_0^b \frac{dx}{1-x^2} + \lim_{b\to 1} \int_b^2 \frac{dx}{1-x^2}$

partial fractions

$\lim_{b\to 1} \int_0^b \frac{1}{(1+x)(1-x)}\,dx = \lim_{b\to 1} \int_0^b \frac{A}{1+x} + \frac{B}{1-x}\,dx$

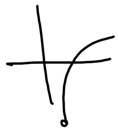
$A(1-x) + B(1+x) = 1$

$\lim_{b\to 1} \frac{1}{2} \int_0^b \frac{1}{1+x} + \frac{1}{1-x}\,dx$

$x = 1: \quad 2B = 1 \quad B = \frac{1}{2}$

$x = -1: \quad 2A = 1 \quad A = 1/2$

$\frac{1}{2}\left(\ln|1+x| - \ln|1-x|\right)$



$\lim_{b\to 1} \frac{1}{2}\left(\ln(1+b) - \ln(1-b) - 0\right)$

$\frac{1}{2}\left(\ln 2 - \cancel{\ln 0}\right)$

diverges

$-\infty$

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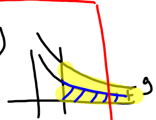
8.4b Improper Integrals

Comparison Test

If $0 \le f(x) \le g(x)$



If $\int_a^\infty g(x)\,dx$ converges

then $\int_a^\infty f(x)\,dx$ also converges

If $\int_a^\infty f(x)\,dx$ diverges

then $\int_a^\infty g(x)\,dx$ also diverges

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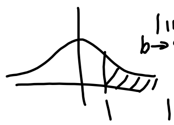
$\int_1^\infty e^{-x^2}\,dx < \int_1^\infty e^{-x}\,dx$

also converges converges

$e^{-x^2} < e^{-x}$

$\frac{1}{e^{x^2}} < \frac{1}{e^x}$



$\lim_{b\to\infty} \int_1^b e^{-x}\,dx$

$\lim_{b\to\infty} \left(-e^{-x}\Big|_1^b\right) = \lim_{b\to\infty} \left(-e^{-b} - -e^{-1}\right) = \frac{1}{e}$

$e^{-b} \to 0$

$e^{-b} = \frac{1}{e^b}$

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Does the integral converge or diverge?

$$\int_1^\infty \frac{dx}{x^5+1} < \int_1^\infty \frac{dx}{x^5} = \lim_{b\to\infty}\int_1^b x^{-5}\,dx$$

Converges   Converges to 1/4

$$\lim_{b\to\infty}\left(\frac{x^{-4}}{-4}\Big|_1^b\right)$$

$$\lim_{b\to\infty}\left(\underset{\downarrow\, 0}{\frac{b^{-4}}{-4}} - \frac{1}{-4}\right) = \frac{1}{4}$$

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Find the volume of the solid obtained by revolving the curve about the x-axis $y = xe^{-x}, 0 \le x < \infty$

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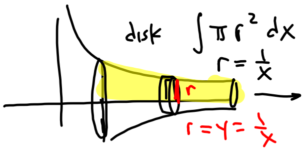
Gabriel's Horn

Consider the region R in the first quadrant bounded above by y=1/x and on the left by x=1. The region is revolved around the x-axis.

a) Show that R has infinite area.
b) Find the volume of the solid.

disk $\int \pi r^2\,dx$

$r = \frac{1}{x}$



a) $\int_1^\infty \frac{1}{x}\,dx$

$$\lim_{b\to\infty}\int_1^b \frac{1}{x}\,dx = \lim_{b\to\infty} \ln x\Big|_1^b = \lim_{b\to\infty} \ln b - \ln 1 = \infty$$

b) $$\int_1^\infty \pi \frac{1}{x^2}\,dx = \lim_{b\to\infty}\int_1^b \pi x^{-2}\,dx$$

$$\lim_{b\to\infty} \pi \frac{x^{-1}}{-1}\Big|_1^b$$

$b^{-1} = \frac{1}{b}$
$\downarrow$
0

$$\lim_{b\to\infty} \pi\left(\underset{\downarrow\, 0}{\frac{b^{-1}}{-1}} - \frac{1}{-1}\right) = \pi$$

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